Constant Terms

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Consider f a (elliptic) modular form of full level and weight k, which has a Fourier expansion given by

$$f(z) = \sum_{n \geq 0} a_n e^{2\pi i n z}.$$

It has the associated automorphic form

$$\tilde{f}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (ci+d)^{-k} f\Big(\frac{ai+b}{ci+d}\Big),$$

on Sp_2. The only non-trivial parabolic P is the one of upper triangular matrices, with Levi and unipotant given respectively

$$M = \begin{pmatrix} m & 0 \\ 0 & m^{-1} \end{pmatrix} \cong \mathrm{GL}_1, \quad N = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \cong \mathbb{G}_a,$$

along which we can now compute the constant term

$$\begin{aligned} \tilde{f}_P(m) &= \int_{N(\mathbb{Z})\backslash N(\mathbb{R})} \tilde{f}(mb) \mathrm{d}b \\ &= \int_{\mathbb{Z}\backslash\mathbb{R}} \tilde{f}\begin{pmatrix} m & mb \\ 0 & m^{-1} \end{pmatrix} \mathrm{d}b \\ &= \int_{\mathbb{Z}\backslash\mathbb{R}} m^k f(m^2 i + m^2 b) \mathrm{d}b \\ &= m^k a_0. \end{aligned}$$